Optimal Output-Trajectory Tracking: Application to Mobile Transporter Avionic Breadboard *

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1 Introduction

Large structures, like manipulators for assembling the space station, are lightweight, and hence flexible. The structural flexibility results in significant elastic vibrations that are caused not only by exogenous perturbations but are also caused by maneuvers like slewing. For example, the Mobile Transporter Unit in the Space Station carries a flexible manipulator with payloads from one part of the Space Station to another. As the payload is moved, induced-vibrations during the movement can lead to undesirable oscillations. Therefore, there is a need for methodologies that allow the movement of such elastic structures along desired output-trajectories while minimizing the induced vibrations. The application of an optimal output-tracking technique to an Avionic Breadboard for the International Space Station’s Mobile Transporter (MT) (see Figure 1) is discussed in this article.

Recent works have solved the output tracking problem using inversion techniques, for example, given a desired output trajectory, inversion-based techniques find input-state trajectories that exactly track the output ([2, 12, 13, 8, 6]). These inversion-based techniques have been successfully applied to the control of multi-joint flexible manipulators in ([17, 18]), and to aircraft control in ([15, 14, 26]). If the number of actuators are the same as the number of tracked-outputs (square system), then the inverse is unique. For a desired output trajectory, the inverse technique finds the unique bounded input-state trajectory, that can achieve exact-tracking. Although such an input-state trajectory exactly tracks the desired output, it might not meet other performance requirements in flexible structures.

Figure 1: Mobile Transporter as it would move on the Space Station (Artist’s Rendering).

For example, during maneuvers of the Mobile Transporter Unit, the structural deformations - determined by the inverse state trajectories - may be unacceptable. Further, excessively large actuator-inputs might be needed during an exact tracking maneuver ([26]). If the number of actuators are more than the number of outputs to be tracked, then the actuator redundancy can be used to optimally minimize the actuator-inputs...
and to reduce structural vibrations ([7]). However, if such actuator redundancy is not available then a compromise is desired between the tracking requirement and other goals like the reduction of internal vibrations and prevention of actuator saturation - i.e., the output trajectory needs to be redesigned.

The problem of redesigning an input to the system to minimize residual and in-maneuver vibrations for flexible structures has been well-studied in literature - see for example ([21, 1, 25, 10, 3, 24]). In tracking problems, however, outputs are usually specified and not the inputs - input trajectories have to be computed from the desired outputs. For nonminimum phase systems (e.g., flexible structures with non-collocated sensors and actuators), the inputs to the system are difficult to determine and require the inversion of the system dynamics. Thus, if an output-maneuver is being designed then typical input-modification based approaches are not directly applicable. Rather than (a) find the input through inversion and then (b) optimize the inverse input, this paper describes a method to directly solve the optimal-inverse problem.

One existing approach to solve the output-redesign problem is to extend the input-redesign problem to the output-redesign problem. Such an approach: (a) chooses a feedback-based tracking controller - the desired output trajectory is now an input to the closed loop system; and (b) redesigns this input to the closed loop system. Thus, the output is redesigned ([21]). These redesigns are, however, dependent on the choice of the tracking controllers ([9]) because the controller optimization and trajectory redesign problems become coupled - this coupled optimization is still an open problem. An additional problem with this approach is that purely feedback-based output tracking may not yield satisfactory tracking due to performance-limitations of feedback-based regulators for nonminimum phase systems ([19]).

In contrast to input-redesign, we use an approach to directly redesign a given output-maneuver \( y_d \) using an optimal-inversion approach [9]. This approach also finds a feedforward input trajectory that achieves exact tracking of the modified output-maneuver. Any errors in the tracking the modified output trajectory, \( y_{opt} \), (due to, for example, initial conditions and modeling errors) can then be corrected using standard feedback approaches, i.e., stabilize the state trajectory, \( x_{opt} \), which exactly tracks the optimal output, \( y_{opt} \) (see, for example, [11]). For example, the feedback-control that stabilizes the state trajectory can be chosen as \( Ke \), where \( e \) is the tracking error and \( K \) is the feedback controller gain. During the exact-inversion-based trajectory redesign, the feedback controller is inactive because the tracking error, \( e \), is zero, and therefore the trajectory redesign is independent of the particular choice of the feedback law (i.e., the choice of \( K \)). In this sense, the optimal-inversion-based output-redesign problem is decoupled from the choice of a particular feedback controller needed to stabilize the system. This approach is illustrated with an example in this paper.

The output trajectory redesign problem is posed as an optimization of a general quadratic cost function as in ([4]) for linear systems. The formulation allows for the minimization of both: (a) residual vibrations and (b) vibrations during the maneuver. Such in-maneuver vibration-reduction is required for tracking maneuvers of flexible spacecraft (see, for example, [4], and recent command-shaping techniques in ([23, 22])). The redesigned output trajectory is obtained by passing the initial output-trajectory through a pre-filter and can be implemented using a convolution similar to the convolutions prevalent in command shaping approaches (see for example [20, 21]).

The criterion for the proposed output-trajectory redesign can be defined in terms of a quadratic cost functional - this cost criterion can be chosen to obtain a trade-off between the precision in tracking, and the reduction of structural vibrations and inputs. For a particular cost criterion, the optimal input can be obtained using a prefilter. An advantage of the present technique is that this prefilter only depends on the choice of the optimization criterion (choice of weighting matrices in a quadratic cost functional) and does not depend on the particular output trajec-
tory. Thus, for a given optimization criterion, the prefilter can be precomputed independent of the output.

If the redesign of a particular output-trajectory is desired, then it is also possible to use the proposed approach for satisfying other criterion—like the prevention of actuator saturation. This can be achieved by manipulating the weighting matrices in the cost functional as in standard linear quadratic optimal control approaches. However, in such redesigns, the appropriate choice of the cost-criterion will depend on the particular output to be tracked. Thus, the resulting output-redesign will also depend on the particular output trajectory.

We begin by illustrating the application of the optimal-inversion technique to the mobile transporter avionic testboard and experimental results are presented. Next, the optimal output-redesign problem is generalized and the solution is presented for general linear systems.

2 Application of Optimal Inversion

The optimal tracking approach was applied to an Avionic Test Breadboard for the Mobile Transporter unit. In the following we describe the Avionic Test Breadboard and present experimental results, which illustrates the output-redesign problem.

2.1 Mobile Transporter

The International Space Station (ISS) Mobile Transporter (MT) is a robotic element, which translates along the Space Station Integrated Truss Assembly (ITA) rails. The MT is a remotely controlled vehicle, which is used to provide mobility for the Mobile Servicing Center (MSC) and its payloads (see Figure 2). The role of the MT is to support the building and maintenance of the space station by translating the MSC and its payloads along the ITA rails between fixed work-sites. In the stationary configuration at the work-site, the MT provides a rigid attachment between the ITA and the MSC by latching down to hard points on the ITA in order to provide a stable platform for the operation of the MSC's large robotic manipulator arm. The initial dynamics analysis and the design of the MT control system can be found in [16].

![Figure 2: Mobile Transporter Servicing Center with Payload.](image)

2.2 MT Avionics Breadboard Test Assembly

The MT avionics control (breadboard) simulator was designed to simulate and verify the operation of the MT control system. The simulator was designed to simulate the key points of the MT drive system [16].

- The fundamental structural natural frequency of the MT and its attached payloads could be as low as 0.1 Hz.
- There is only one co-located observer with the actuator and the states of the payload is not measured.
- There is significant backlash in the drive system.

An avionics control breadboard test assembly (see Figure 3) has been set up to serve as a simplified MT model for control analysis and simulation. The simulator is made of a large adjustable
flywheel (inertia) representing the payload connected to a small flywheel (inertia) representing the MT with a flexible shaft to simulate the MT and its payload with diverse natural frequencies.

Figure 3: Mobile Transporter Avionics Control Breadboard Test Assembly

The simulator lowest natural frequency is about 0.1 Hz. The drive motor (brush-less DC) is connected to the gearbox with a coupler which has adjustable backlash capability to simulate the MT drive system backlash. The gearbox output is connected to the small inertia. There is an 8,000 lines/revolution encoder for the drive motor position, and two 64,000 lines/revolution encoders for each of the two flywheels. The encoders on the flywheels are only used to monitor their states and are not used in the control feedback, since there is only one observer in the MT drive system and the state of the payloads are not monitored during the MT translation.

The goal of choosing an output trajectory is to achieve a fast maneuver without saturating the inputs and without causing too much internal vibrations. Figure 4 presents experimental results for tracking an output trajectory – the output is the angular position of the large adjustable flywheel in the avionics control breadboard test assembly. Note that the desired output is tracked well (the maximum tracking error is less than 1% of the maximum rotation) as seen in Figure 4. Also shown is the structural vibrations, i.e., difference υ between the angular positions of the large and small flywheels (shown in Figure 3).

Since the desired trajectory was chosen using a bang-bang type acceleration profile, it might be possible to re-design this output trajectory to reduce the structural vibrations. This redesign problem can be cast as an optimization problem of the form

\[ \int_{-\infty}^{\infty} \left\{ u(t)^T R u(t) + v(t)^T Q v(t) + [e_d(t)]^T Q_e [e_d(t)] \right\} dt \]

where \( R, Q_e \) and \( Q_v \) represents the weight on control input \( u \), structural vibration \( v \), and the error in output tracking \( e_d \) respectively.

Figure 4: Experimental Results. Desired and achieved outputs are compared at the top, the middle plot shows the error \( e_d \) in tracking, and the bottom plot shows the structural vibration. The horizontal axis is time in seconds.

The modified output and tracking results are shown in Figure 5. Note that the modification trajectory achieves the same output transition within a similar amount of time (25 seconds). However, the structural vibrations are sig...
significantly reduced (more than 30% lower in magnitude).

### 3 Problem Formulation

**System Inversion for Exact Tracking**

Next, we present a generalization of the methodology in the last Section. Consider a single-input single-output linear system described by

\[
M \ddot{x}(t) + D \dot{x}(t) + Kx(t) = Bu(t) \\
y(t) = Cx(t)
\]

where \( x \in \mathbb{R}^n \) represents the generalized displacements, \( u \in \mathbb{R} \) is the input and \( y \in \mathbb{R} \) is the output. \( M, D \) and \( K \) are the mass, damping and stiffness matrices. These equations can be represented in the frequency domain as

\[
[-\omega^2 M + j\omega D + K] X_a(j\omega) = BU(j\omega) \\
Y_a(j\omega) = CX_a(j\omega)
\]

where \( X_a, U, Y_a \) are the Fourier transforms of the vector \( x \), the input \( u \) and the output \( y \), respectively. A result for optimization for general multi-input multi-output systems can be found in [9]. In the following, we restrict our discussion to linear time-invariant single-input single-output systems. The transfer functions from input to state and input to output can be written as

\[
X_a(j\omega) = [-\omega^2 M + j\omega D + K]^{-1} BU(j\omega) \\
:= G_X(j\omega) U(j\omega) \\
Y_a(j\omega) = CG_X(j\omega)U(j\omega) \\
:= G_Y(j\omega) U(j\omega)
\]

**The Performance Criterion**

Trajectory redesign seeks a compromise between the goal of tracking the desired trajectory \( y_d \) and other goals like reducing the magnitude of input and vibrations. We formulate this redesign problem as the minimization of a quadratic cost function of the type

\[
\int_{-\infty}^{\infty} \left\{ u(t)^T Ru(t) + x(t)^T Q_x x(t) + \\
y(t) - y_d(t))^T Q_y [y(t) - y_d(t)] \right\} dt
\]

where \( R, Q_x \) and \( Q_y \) represents the weight on control input, structural vibration, and the error in output tracking respectively. Using Parseval’s theorem we rewrite our optimization problem in frequency domain as the minimization of the cost function

\[
\int_{-\infty}^{\infty} \left\{ (U(j\omega))^* R U(j\omega) + X_a(j\omega)^* Q_x X_a(j\omega) + \\
[Y_a(j\omega) - Y_{a,d}(j\omega)]^* Q_y [Y_a(j\omega) - Y_{a,d}(j\omega)] \right\} d\omega
\]

where the superscript * denotes complex conjugate transpose.

![Figure 5: Experimental Results with modified trajectory. Desired and achieved outputs are compared at the top, the middle plot shows the error \( e_a \) in tracking, and the bottom plot shows the structural vibration. The horizontal axis is time in seconds.](image-url)
which shows that the optimal output-trajectory redesign can be described as a pre-filter, which maps desired output trajectories, \( y_d \), to the redesigned output trajectory, \( y_{opt} \). This pre-filter, \( G_f \), does not depend on the particular choice of desired trajectory and hence can be pre-computed. A result for general Multi-input Multi-output systems can be found in [9].

**Lemma**

- The modified output trajectory, \( y_{opt} \), is given by
  \[
  Y_{opt}(j\omega) = G_f(j\omega)Y_d(j\omega)
  \]
  where
  \[
  G_f = G_{y_x}G_y^0[R + G_{y_x}Q_xG_{y_x} + G_{y_x}Q_yG_{y_x}].
  \]

- The input trajectory, \( u_{opt} \), that achieves exact tracking of the modified output trajectory is given by
  \[
  U_{opt}(j\omega) = G_u(j\omega)Y_d(j\omega)
  \]
  where
  \[
  G_u = G_{y_x}^{-1}G_f
  \]

Note that the dependence on \( j\omega \) is not explicitly written for compactness.

**Proof:** This follows from arguments in [9]. \( \square \)

We point out two extreme cases. First, if the weight on the tracking error is zero (\( Q_y = 0 \), and \( R \) positive definite) then the best strategy is not to track the desired trajectory at all. Second, if the weight on the inputs and states are zero (\( R = 0 \), \( Q_x = 0 \), and \( Q_y \) positive definite) then \( y_{opt} = y_d \). This implies that exact tracking is optimal, and the cost is again zero.

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**References**


